Comments on "Tracer-Spark Technique for Velocity Mapping of Hypersonic Flow Fields"

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In a recent note, Kyser[1] suggests a method to determine the density distribution in hypersonic wakes involving measurement of the local flow velocities. The velocities are to be measured by a variation of Bomelburg's technique (Ref. 1 of Kyser's paper) and are based on photographs of a rapid sequence of precisely timed electric sparks between two parallel rod electrodes. Since each spark tends to follow the ionized path of the preceding discharge, the velocity distribution can be found from the displacement and distortion of the spark columns shown on photographic records. It is the purpose of these comments to point out a potential source of errors of such measurements.

It was observed[2] that a spark column follows the gas flow exactly only if the flow velocity is constant, and that it moves faster than the gas if the flow is accelerating. The column velocity may exceed the gas velocity by as much as 60% if the acceleration is produced by a shock wave. This behavior of the spark-heated column may be explained by the difference of the density of the column and that of the surrounding gas. One may consider the flow accelerations as being caused by an equivalent gravitational field; the net force acting on the column is then the difference between its weight in this field and the buoyancy force that is equal to the corresponding weight of surrounding gas displaced by the column. As a result of its motion relative to the gas, the column is practically instantaneously transformed into a vortex, and, although this transformation consumes a large fraction of the energy of the relative motion, sufficient energy is left to cause an appreciable residual velocity difference.

A theory for the dynamics of a small gaseous region having a different density than the surrounding gas shows, in agreement with experimental observations of bubbles of known density ratio, that the ratio of a change of the gas velocity to the corresponding change of the bubble velocity is constant regardless of the magnitude of the acceleration.[2] This result applies only if the density ratio is constant and does not account for changes of the effective density ratio by cooling and diffusion. Because of these effects, the excess velocity of a spark column over the gas velocity behind a shock wave is reduced from its initial value of about 60 to 40% if the average velocity is determined for an interval of about 50 μsec.

Since these errors apply only to the accelerated part of the gas flow, the over-all error in a particular flow may be considerably smaller than the values indicated in the foregoing and may even be negligible in some cases. In Fig. 1, let the heavy solid line represent the motion of the gas in the position-time plane, and place the origin of the coordinates at the point corresponding to a spark discharge. If the next spark had been discharged after a time interval Δt and the gas had continued to flow at its initial velocity V_0, it would have traveled the distance $V_0\,\Delta t$, and the spark column would have traveled exactly the same distance. In the accelerating flow shown in the figure, the gas travels the distance Δx, whereas the spark column, traveling along the broken line, travels Δx_S. According to the preceding discussion, the ratio of the accelerated parts of the motion is equal to a value α, which depends somewhat on the time interval between the

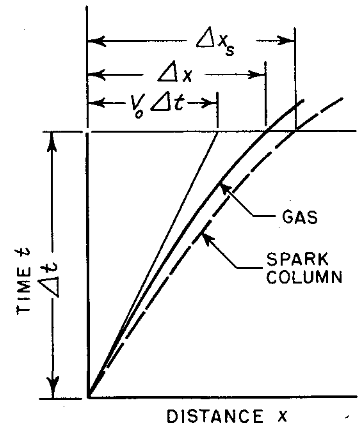

Fig. 1 Position-time plot of an accelerating gas flow and the corresponding motion of a spark column.

sparks and possibly also on the experimental conditions, so that

$$(\Delta x_S - V_0\Delta t)/(\Delta x - V_0\Delta t) = \alpha \tag{1}$$

According to the foregoing, the value of α is approximately 1.4 for a typical interval $\Delta t = 50$ μsec and for the experimental conditions of Ref. 2 (air at near-atmospheric pressure, a spark gap of about 1 cm, and a spark produced by a 1-μf capacitor charged to 400 v and discharged through an automobile spark coil). The average gas velocity is given by $V = \Delta x/\Delta t$ and the velocity measured by observation of the spark column by $V_S = \Delta x_S/\Delta t$. Equation (1) therefore yields the velocity error as

$$V_S - V = (\alpha - 1)(V - V_0) \tag{2}$$

This result shows that the velocity error is approximately equal to 40% of the velocity change between consecutive sparks. In the empty test section of a hypersonic tunnel, the velocity may change less than 1% in the distance that the gas moves during 50 μsec, and the error of the velocity measurement then would be less than $\frac{1}{2}$%. However, the corresponding velocity change in the wake of a model may well exceed 10%, resulting in a velocity error of more than 4%. Since the gas density is essentially proportional to $1/V^2$, the density error would then exceed 8%.

These remarks do not mean that the technique of "tagging" a flow with a spark discharge is unsuitable for velocity measurements, but that the behavior of the spark columns should be considered in each case in order to ascertain that errors caused by flow accelerations do not exceed tolerable limits.

References

[1] Kyser, J. B., "Tracer-spark technique for velocity mapping of hypersonic flow fields," AIAA J. 2, 393–394 (1964).
[2] Rudinger, G. and Somers, L. M., "Behavior of small regions of different gases carried in accelerated gas flows," J. Fluid Mech. 7, 161–176 (1960).

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Reply by Author to G. Rudinger

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In the preceding comment, Rudinger analyzes the effects on density measurements of a velocity error of 40% of the velocity change between consecutive sparks. The factor α, which is used in the analysis, is a measure of this error and is given in Ref. 1 as 1.4 for a particular experiment. In Ref. 1,

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a theory is developed which allows the computation of α as a function of density ratio (spark-column density divided by freestream density) and shape factor. From the results of this theory and from a comparison of other data, where a column of light gas is substituted for the spark-heated gas, presented in Ref. 1, it may be seen that the value $\alpha = 1.4$ corresponds roughly to the case where the density ratio is zero.

From this discussion it may be surmised that Rudinger has used an extreme case, involving considerable heating of the spark column, to demonstrate the significance of the possible error. At Stanford, emphasis has been placed upon the reduction of energy added to the flow, both by reducing the size of the storage capacitor and by inserting damping resistors. The results show that satisfactory spark photographs can be obtained at spark energy levels several orders of magnitude below those required to achieve a discernible, i.e., Mach number 2, blast wave from the spark. Thus, the density change resulting from the spark does not need to be great.

In conclusion, it is the author's opinion that Rudinger did not properly identify the possible error in velocity measurement with the density ratio. The point that must be made is that care should be taken to insure that a minimum amount of energy is added to the gas in the spark column in order to insure a minimum discrepancy between column velocity and local stream velocity.

Reference

[1] Rudinger, G. and Somers, L. M., "Behavior of small regions of different gases carried in accelerated gas flows," J. Fluid Mech. **7**, 161–176 (1960).

Comment on "Effective Shear Modulus of Honeycomb Cellular Structure"

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In a recent paper,[1] Penzien and Didriksson have presented an interesting analysis of honeycomb cellular structures. However, there is a very disturbing lack of references in this paper, which leads one to have the impression that this work represents the first venture into this field of analysis. It must be pointed out that this particular topic was comprehensively dealt with in a paper[2] published in 1958. Reference 2, which is based on work reported by Ref. 3, presents theoretical analyses, a survey of experimental methods, and test data. The complete stiffness and flexibility matrices were derived for a honeycomb sandwich subjected to shear loading in any direction in the plane of the sandwich. By means of the dual principles of the unit load and the unit displacement methods,[4] the upper and lower limits on the shear modulus were clearly delineated and determined. The unit load method underestimates the stiffness of the structure; the unit displacement method overestimates the stiffness.

Although it has not been explicitly stated, the authors have chosen to develop a formula for the shear modulus of the honeycomb core on the basis of the unit load approach. This development includes a correction factor to account for the effect of warping restraint within the core. The computation of the correction factor is somewhat complex, however, and it has, in fact, been neglected in the presentation of the final equation, Eq. (44), which is thereby identical to Eq. (2.20) of Ref. 2.

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It is to be emphasized, therefore, that this equation is an approximation to the exact solution and represents an underestimation of the stiffness, i.e., a "lower bound." To obtain an upper bound, the analyst can employ Eq. (2.21) of Ref. 2 which can be stated, in the notation of Ref. 1, as

$$\frac{G_c}{G} = \frac{\sin\theta(R + \cos^2\theta)}{(a/t)(R + \cos\theta)[(R + \cos^2\theta)\sin^2\phi + \sin^2\theta\cos^2\phi]} \quad (1)$$

In view of the complexity of the correction factor required for an exact formulation, the writer feels that the concept of deriving, in a simple fashion, upper and lower bounds to the exact solution has considerable merit.

Also included in the 1958 paper are design curves for honeycomb cores and a brief study of the anisotropy of the honeycomb. The latter led to a suggested type of honeycomb possessing plane isotropy. In addition, a shear efficiency parameter for use in comparison of various types of cores was developed. Although the formulas derived are specifically for hexagonal core shapes, the methods are, naturally, directly applicable to any other built-up foil cores.

The experimental method of determining the shear modulus described by Penzien and Didriksson is of interest. Unfortunately, the nature of the test frame is such that the sandwich so tested is not really typical of the type used in practice. This is felt to be a fairly important point. The actual test data derived using this method have been omitted, and only a brief comment as to the accuracy has been included. A number of standard methods that could be applied to realistic core-face combinations were critically reviewed in the older work and a simple method selected which gave excellent agreement with theoretical values. This was a straightforward three-point bend test in which the small shear deflection component was determined through an accurate interpretation of the results by means of standard statistical procedures. For such a method, neither special test specimens nor testing machines are required.

Another promising method of determining the shear modulus is the so-called five-point bending test developed by Howard.[5] Here, the bending deflection component is eliminated, and only a shear deflection remains. No test results have been published for this scheme, however.

An important point must be made in connection with this particular problem. It is well known that it is very difficult to measure the shear modulus of a sandwich core, especially one of the built-up honeycomb type. Indeed, a tremendous amount of effort has gone into experimental work in this field, with varied results. The reason is that the shear deformation effects are very small and cannot be measured accurately, since they usually occur in association with more dominant bending deflections. Conversely, it appears reasonable for all practical purposes that it is not necessary to know the value of the shear modulus to any great degree of accuracy. Thus there is no great need for highly sophisticated methods of testing to determine the shear modulus. Moduli computed using formulas such as those given in the papers under discussion, which are based upon known properties and dimensions of the core structure, should be sufficient for all purposes.

References

[1] Penzien, J. and Didriksson, T., "Effective shear modulus of honeycomb cellular structure," AIAA J. 2, 531–535 (1964).

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